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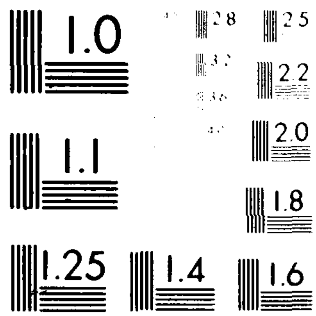

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INCOMPLETE BLOCK DESIGNS FOR COMPARING TREATMENTS WITH A CONTROL (IV): OPTIMAL DESIGNS FOR $p = 4$, $k = 4$

by

Robert E. Bechhofer
Cornell University

Ajit C. Tamhane
Northwestern University

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ABSTRACT

The present paper continues the study of balanced treatment incomplete block (BTIB) designs initiated in [1], [2], and [3]. This class of designs was proposed for the problem of comparing simultaneously $p \geq 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p + 1$. A list of generator designs, the conjectured minimal complete class of generator designs, a catalog of admissible designs, and tables of optimal designs are given for $p = 4$, $k = 4$. Some comparisons are made with admissible designs for $p = 4$, $k = 3$.

Key words and phrases: Multiple comparisons with a control, balanced treatment incomplete block (BTIB) designs, admissible designs, S-inadmissible designs, C-inadmissible designs, minimal complete class of generator designs, optimal designs.

1. INTRODUCTION

The present paper continues the study of balanced treatment incomplete block (BTIB) designs begun in [1], [2] and [3]. This class of designs was proposed for the problem of comparing simultaneously $p \geq 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p + 1$. Papers [1]-[3] give the background, motivation and notation for this study.

In [1] a general theory of BTIB designs was developed; in [2] optimal designs were given for the cases $p = 2$, $k = 2(1)6$ and $p = 3$, $k = 3$ while in [3] optimal designs were given for the cases $p = 4$, $k = 3$ and $p = 5$, $k = 3$. In the present paper we give optimal designs for the case $p = 4$, $k = 4$; these optimal designs are subject to the same qualification as those given in [3]--namely that they are optimal relative to the generator designs known to us. However, we conjecture (as we did for $p = 3$, $k = 4$ and $p = 3$, $k = 5$ in [3]) that we have enumerated all of the admissible generator designs for $p = 4$, $k = 4$, and that if additional ones do exist the incremental gain achieved by using the full set in place of our set would be very small.

In our study of the cases $p = 4$, $k = 3$ and $p = 5$, $k = 3$ it was necessary for us to generalize and develop further certain concepts which we had introduced for the cases $p = 2$, $k = 2(1)6$ and $p = 3$, $k = 3$. For our present study of the case $p = 4$, $k = 4$ no further generalizations were required. (See, however, Remark 2.3 of the present paper.) Thus the reader is referred to [3] for the definitions of inadmissibility, S-inadmissibility and C-inadmissibility used in this paper. In presenting our results for $p = 4$, $k = 4$ we hope to accomplish two objectives: (a) To provide other researchers

in the combinatorial design area with our list of generator designs and our conjectured minimal complete class of generator designs with the hope that they can supply additional generator designs (if any exist), and more importantly that they can propose a feasible method or methods of constructing an exhaustive set of such designs, and (b) To provide experimenters with optimal (or nearly optimal) designs that can be implemented in practice.

The reader is referred to Sections 2 and 3 of [2] and Sections 1 and 2 of [3] for an exact statement of the multiple comparison problem under consideration, expressions for the BLUE's of the treatment effect differences $\alpha_0 - \alpha_i$ $(1 \leq i \leq p)$, their variances and correlations, and an expression for the confidence coefficient P associated with joint one-sided confidence interval estimates of the $\alpha_0 - \alpha_i$ $(1 \leq i \leq p)$.

2. RESULTS FOR $p = 4$, $k = 4$

2.1 List of generator designs

The generator designs that we have constructed for $p = 4$, $k = 4$ (by the methods described in Section 3.2 of [1], or by other methods) are listed in Table 2.1. As in [3] we have not exhibited equivalent designs which differ only trivially from those given in the tables.

For the generator designs in Table 2.1 we note that: a) D_5 is S-inadmissible w.r.t. D_1, b) D_6 is equivalent to $D_3 \cup D_4$, c) D_7 is S-inadmissible w.r.t. D_3 and $D_3 \cup D_4$, d) D_8 is S-inadmissible w.r.t. $D_3 \cup 2D_4$, e) D_9 is S-inadmissible w.r.t. $D_1 \cup D_3$, f) D_{10} is S-inadmissible w.r.t. $D_1 \cup D_3 \cup 2D_4$, g) D_{11} is S-inadmissible w.r.t. $6D_4$, h) D_{12} is S-inadmissible w.r.t. $12D_4$, and i) D_{13} is S-inadmissible

Table 2.1

Generator Designs for $p = 4$, $k = 4$

Label	Design	b_i	$\lambda_0^{(i)}$	$\lambda_1^{(i)}$
D_1	$\begin{Bmatrix} 0&0&0&0 \\ 1&1&1&2 \\ 2&2&3&3 \\ 3&4&4&4 \end{Bmatrix}$	4	3	2
D_2	$\begin{Bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 1&2&3&4 \\ 1&2&3&4 \end{Bmatrix}$	4	4	0
D_3	$\begin{Bmatrix} 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 1&1&1&2&2&3 \\ 2&3&4&3&4&4 \end{Bmatrix}$	6	6	1
D_4	$\begin{Bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{Bmatrix}$	1	0	1
D_5	$\begin{Bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 1&2&3&4 \end{Bmatrix}$	4	3	0
D_6	$\begin{Bmatrix} 0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&2 \\ 1&1&2&0&2&3&3 \\ 2&4&4&3&3&4&4 \end{Bmatrix}$ $\begin{Bmatrix} 0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&3 \\ 1&1&2&2&2&2&4 \\ 3&4&3&4&3&4&4 \end{Bmatrix}$	7	6	2
D_7	$\begin{Bmatrix} 0&0&0&0&0&0&0 \\ 2&0&0&0&0&0&0 \\ 3&1&1&1&0&0&0 \\ 4&2&3&4&2&3&4 \end{Bmatrix}$	7	6	1

Table 2.1 (continued)

Label	Design	b_i	$\lambda_0^{(i)}$	$\lambda_1^{(i)}$
D_8	$\left\{\begin{matrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&2&3 \\ 1&1&2&4&2&4&4&4 \\ 2&3&3&4&3&4&4&4 \end{matrix}\right\}$	8	6	2
D_9	$\left\{\begin{matrix} 0&0&0&0&0&0&0&0&0&0 \\ 1&2&0&0&0&0&0&0&0&0 \\ 3&3&1&1&1&1&2&2&0&0 \\ 4&4&2&2&3&4&3&4&3&4 \end{matrix}\right\}$	10	9	2
D_{10}	$\left\{\begin{matrix} 0&0&0&0&0&0&0&0&0&0&0&0 \\ 1&1&1&2&2&2&3&3&3&4&4&4 \\ 1&1&1&2&2&2&3&3&3&4&4&4 \\ 2&3&4&1&3&4&1&2&4&1&2&3 \end{matrix}\right\}$	12	9	4
D_{11}	$\left\{\begin{matrix} 1&1&1&2&2&3 \\ 1&1&1&2&2&3 \\ 2&3&4&3&4&4 \\ 2&3&4&3&4&4 \end{matrix}\right\}$	6	0	4
D_{12}	$\left\{\begin{matrix} 1&1&1&2&2&2&3&3&3&4&4&4 \\ 2&3&4&1&3&4&1&2&4&1&2&3 \\ 2&3&4&1&3&4&1&2&4&1&2&3 \\ 2&3&4&1&3&4&1&2&4&1&2&3 \end{matrix}\right\}$	12	0	6
D_{13}	$\left\{\begin{matrix} 1&1&1&1&2&2&2&2&3&3&3&3&4&4&4&4 \\ 1&1&1&2&1&1&1&2&1&1&1&2&1&1&1&2 \\ 2&2&3&3&2&2&3&3&2&2&3&3&2&2&3&3 \\ 3&4&4&4&3&4&4&4&3&4&4&4&3&4&4&4 \end{matrix}\right\}$	16	0	10

w.r.t. $16D_4$. Thus for $p = 4$, $k = 4$ and $b \geq 4$ it suffices to consider unions of replications of D_1, D_2, D_3, D_4 when seeking the optimal design for a specified d/σ. We conjecture that $\{D_1, D_2, D_3, D_4\}$ is the minimal complete class of generator designs for $p = 4$, $k = 4$.

2.2 Catalog of admissible designs

A catalog of admissible designs has been prepared based on the set of admissible generator designs given for $p = 4$, $k = 4$ in Table 2.1. This catalog is given in Table 2.2 for $b = 4(1)27$. It is to be noted that the number of admissible designs increases rapidly with b for $p = 4$, $k = 4$; e.g., for $b = 27$ we have 13 admissible designs for $p = 4$, $k = 4$ whereas for $b = 27$ we had (see [3]) 2 admissible designs for $p = 4$, $k = 3$ and 5 admissible designs for $p = 5$, $k = 3$.

Remark 2.1: We note from Table 2.2 that when D_3 appears as an admissible design for a particular b, it is always associated with the smallest value of τ^2 and ρ for that b; hence, the associated design is always optimal for that b for d/σ sufficiently large.

2.3 Tables of optimal designs

Optimal designs for $p = 4$, $k = 4$ are given in Table 2.3 for $d/\sigma = 0.1(0.1)1.0$ and $b = 4(1)31$. Optimal designs that achieve a specified confidence coefficient $1 - \alpha$ are given as a function of d/σ for $1 - \alpha = 0.75, 0.80, 0.85, 0.90, 0.95$ and 0.99 in Table 2.4. These designs were found by a complete computer search among all admissible designs.

Remark 2.2: We note from Table 2.4 that for $d/\sigma \to 0$ and $1 - \alpha \to 1$ the optimal design essentially employs only replications of D_1 which is a BIB design among the 4 test treatments augmented by a control treat-

ment in each block. (An analogous phenomenon was reported for $p = 4$, $k = 3$ and $p = 5$, $k = 3$ in Remark 3.4 of [3].)

Remark 2.3: As mentioned in Remark 2.1 of [3] it may be of some interest to compare designs with different k-values for the same p-value. Thus an experimenter who is faced with the choice of the block size (subject to the restriction that the common block size $k < p + 1$) may wish to make such broader comparisons (which we have not made before), and rule out inadmissible designs using the following generalized definition: If D_1 and D_2 are BTIB designs with parameters $(b_1,k_1,\tau_1^2,\rho_1)$ and $(b_2,k_2,\tau_2^2,\rho_2)$, respectively, with $N_1 = k_1b_1 \leqq N_2 = k_2b_2$, $\tau_1^2 \leqq \tau_2^2$, and $\rho_1 \geqq \rho_2$ with at least one inequality strict then D_2 is said to be inadmissible w.r.t. D_1. (This definition is equivalent to the one given in Remark 2.1 of [3] as a consequence of Theorem 5.1 of [1].)

Using this definition it can be verified that all designs for $p = 4$, $k = 3$ and $b = 16$ (see Table 3.3 in [2]) are inadmissible w.r.t. some design for $p = 4$, $k = 4$ and $b = 12$ (see Table 2.1 of the present paper), the N being equal to 48 in both cases. Similarly, all designs for $p = 4$, $k = 3$ and $b = 20, 24, 28, 32$ are inadmissible w.r.t. some design for $p = 4$, $k = 4$ and $b = 15, 18, 21, 24$, respectively. We conjecture that the same phenomenon occurs for higher values of b. This indicates that for given p and fixed $kb = N$, designs with larger k-values $(k < p + 1)$ are preferable since such designs are "more complete."

We have obtained one example in which a BTIB design for $p = 4$, $k = 4$ is inadmissible (in the broader sense) w.r.t. a BTIB design for $p = 4$, $k = 3$. (Note: This pair of designs was found by comparing all admissible designs for $p = 4$, $k = 4$, $b = 4$-75 with all admissible designs for $p = 4$, $k = 3$, $b = 4$-100, and was the only such design found.) If we denote by $D_i(3)$

$(1 \leq i \leq 5)$ and $D_i(4)$ $(1 \leq i \leq 4)$ the designs for $p = 4$ in Table 3.1 of [3] and Table 2.1 of [4], respectively, we consider the designs

$$D_2(3) \cup D_5(3) = \left\{ \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 & 3 & 2 & 2 & 3 & 3 \\ 2 & 3 & 4 & 3 & 4 & 4 & 3 & 4 & 4 & 4 \end{matrix} \right\}$$

with $b = 10$, $\lambda_0 = 3$, $\lambda_1 = 3$, and

$$D_2(4) \cup 4D_4(4) = \left\{ \begin{matrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 4 & 3 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 & 4 & 4 & 4 \end{matrix} \right\}$$

with $b = 8$, $\lambda_0 = 4$, $\lambda_1 = 4$. Both designs have $\tau^2 = 4/10$, $\rho = 1/2$ and therefore achieve the same joint confidence coefficient. However, the design for $k = 4$ has $N = 32$ while the design for $k = 3$ has $N = 30$. Thus, although each design is admissible for its own k-value, the design for $k = 4$ is inadmissible w.r.t. the design for $k = 3$. (This result is perhaps not too surprising here since the design for $k = 3$ is more balanced than the one for $k = 4$. In fact, the design for $k = 3$ is optimal for d/σ not too large whereas the design for $k = 4$ is not optimal for any d/σ.)

3. ACKNOWLEDGMENT

We are happy to acknowledge the assistance of Mr. Stephen Mykytyn who computed the tables given in this paper, and who wrote the computer program which detected the interesting example described above.

Table 2.2

Catalog of admissible designs[1/] for $p = 4$, $k = 4$

No. of blocks (b)	D_1 $b_1 = 4$ $\lambda_0^{(1)} = 3$ $\lambda_1^{(1)} = 2$	D_2 $b_2 = 4$ $\lambda_0^{(2)} = 4$ $\lambda_1^{(2)} = 0$	D_3 $b_3 = 6$ $\lambda_0^{(3)} = 6$ $\lambda_1^{(3)} = 1$	D_4 $b_4 = 1$ $\lambda_0^{(4)} = 0$ $\lambda_1^{(4)} = 1$	λ_0	λ_1	τ^2	ρ
4	1	0	0	0	3	2	0.6061	0.400
5	1	0	0	1	3	3	0.5333	0.500
6	1	0	0	2	3	4	0.4912	0.571
	0	0	1	0	6	1	0.4667	0.143
7	1	0	0	3	3	5	0.4638	0.625
	0	1	0	3	4	3	0.4375	0.429
	0	0	1	1	6	2	0.3810	0.250
8	1	0	0	4	3	6	0.4444	0.667
	0	1	0	4	4	4	0.4000	0.500
	2	0	0	0	6	4	0.3030	0.400
9	1	0	0	5	3	7	0.4301	0.700
	0	1	0	5	4	5	0.3750	0.556
	2	0	0	1	6	5	0.2821	0.455
10	1	0	0	6	3	8	0.4190	0.727
	0	1	0	6	4	6	0.3571	0.600
	2	0	0	2	6	6	0.2667	0.500
	1	0	1	0	9	3	0.2540	0.250
11	1	0	0	7	3	9	0.4103	0.750
	0	1	0	7	4	7	0.3438	0.636
	2	0	0	3	6	7	0.2549	0.538
	1	1	0	3	7	5	0.2540	0.417
	1	0	1	1	9	4	0.2311	0.308
12	1	0	0	8	3	10	0.4031	0.769
	0	1	0	8	4	8	0.3333	0.667
	2	0	0	4	6	8	0.2456	0.571
	1	1	0	4	7	6	0.2396	0.462
	3	0	0	0	9	6	0.2020	0.400

[1/] For each number of blocks, the number under D_i $(1 \leq i \leq 4)$ in the body of the table is the frequency f_i with which D_i appears in the design $D = \bigcup_{i=1}^{4} f_i D_i$.

Table 2.2 (continued)

No. of blocks (b)	D_1 $b_1 = 4$ $\lambda_0^{(1)} = 3$ $\lambda_1^{(1)} = 2$	D_2 $b_2 = 4$ $\lambda_0^{(2)} = 4$ $\lambda_1^{(2)} = 0$	D_3 $b_3 = 6$ $\lambda_0^{(3)} = 6$ $\lambda_1^{(3)} = 1$	D_4 $b_4 = 1$ $\lambda_0^{(4)} = 0$ $\lambda_1^{(4)} = 1$	λ_0	λ_1	τ^2	ρ
13	1	0	0	9	3	11	0.3972	0.786
	0	1	0	9	4	9	0.3250	0.692
	2	0	0	5	6	9	0.2381	0.600
	1	1	0	5	7	7	0.2286	0.500
	3	0	0	1	9	7	0.1922	0.438
14	1	0	0	10	3	12	0.3922	0.800
	0	1	0	10	4	10	0.3182	0.714
	2	0	0	6	6	10	0.2319	0.625
	1	1	0	6	7	8	0.2198	0.533
	3	0	0	2	9	8	0.1843	0.471
	2	0	1	0	12	5	0.1771	0.294
15	1	0	0	11	3	13	0.3879	0.813
	0	1	0	11	4	11	0.3125	0.733
	2	0	0	7	6	11	0.2267	0.647
	1	1	0	7	7	9	0.2126	0.563
	3	0	0	3	9	9	0.1778	0.500
	2	0	1	1	12	6	0.1667	0.333
16	1	0	0	12	3	14	0.3842	0.824
	0	1	0	12	4	12	0.3077	0.750
	2	0	0	8	6	12	0.2222	0.667
	1	1	0	8	7	10	0.2067	0.588
	3	0	0	4	9	10	0.1723	0.526
	2	1	0	4	10	8	0.1714	0.444
	4	0	0	0	12	8	0.1515	0.400
17	1	0	0	13	3	15	0.3810	0.833
	0	1	0	13	4	13	0.3036	0.765
	2	0	0	9	6	13	0.2184	0.684
	1	1	0	9	7	11	0.2017	0.611
	3	0	0	5	9	11	0.1677	0.550
	2	1	0	5	10	9	0.1652	0.474
	4	0	0	1	12	9	0.1458	0.429

No. of blocks (b)	D_1 $b_1 = 4$ $\lambda_0^{(1)} = 3$ $\lambda_1^{(1)} = 2$	D_2 $b_2 = 4$ $\lambda_0^{(2)} = 4$ $\lambda_1^{(2)} = 0$	D_3 $b_3 = 6$ $\lambda_0^{(3)} = 6$ $\lambda_1^{(3)} = 1$	D_4 $b_4 = 1$ $\lambda_0^{(4)} = 0$ $\lambda_1^{(4)} = 1$	λ_0	λ_1	τ^2	ρ
18	1	0	0	14	3	16	0.3781	0.842
	0	1	0	14	4	14	0.3000	0.778
	2	0	0	10	6	14	0.2151	0.700
	1	1	0	10	7	12	0.1974	0.632
	3	0	0	6	9	12	0.1637	0.571
	2	1	0	6	10	10	0.1600	0.500
	4	0	0	2	12	10	0.1410	0.455
	3	0	1	0	15	7	0.1364	0.318
19	1	0	0	15	3	17	0.3756	0.850
	0	1	0	15	4	15	0.2969	0.789
	2	0	0	11	6	15	0.2121	0.714
	1	1	0	11	7	13	0.1937	0.650
	3	0	0	7	9	13	0.1603	0.591
	2	1	0	7	10	11	0.1556	0.524
	4	0	0	3	12	11	0.1369	0.478
	3	0	1	1	15	8	0.1305	0.348
20	1	0	0	16	3	18	0.3733	0.857
	0	1	0	16	4	16	0.2941	0.800
	2	0	0	12	6	16	0.2095	0.727
	1	1	0	12	7	14	0.1905	0.667
	3	0	0	8	9	14	0.1573	0.609
	2	1	0	8	10	12	0.1517	0.545
	4	0	0	4	12	12	0.1333	0.500
	5	0	0	0	15	10	0.1212	0.400
21	1	0	0	17	3	19	0.3713	0.864
	0	1	0	17	4	17	0.2917	0.810
	2	0	0	13	6	17	0.2072	0.739
	1	1	0	13	7	15	0.1876	0.682
	3	0	0	9	9	15	0.1546	0.625
	2	1	0	9	10	13	0.1484	0.565
	4	0	0	5	12	13	0.1302	0.520
	3	1	0	5	13	11	0.1296	0.458
	5	0	0	1	15	11	0.1175	0.423

Table 2.2 (continued)

No. of blocks (b)	D_1 $b_1 = 4$ $\lambda_0^{(1)} = 3$ $\lambda_1^{(1)} = 2$	D_2 $b_2 = 4$ $\lambda_0^{(2)} = 4$ $\lambda_1^{(2)} = 0$	D_3 $b_3 = 6$ $\lambda_0^{(3)} = 6$ $\lambda_1^{(3)} = 1$	D_4 $b_4 = 1$ $\lambda_0^{(4)} = 0$ $\lambda_1^{(4)} = 1$	λ_0	λ_1	τ^2	ρ
22	1	0	0	18	3	20	0.3695	0.870
	0	1	0	18	4	18	0.2895	0.818
	2	0	0	14	6	18	0.2051	0.750
	1	1	0	14	7	16	0.1851	0.696
	3	0	0	10	9	16	0.1522	0.640
	2	1	0	10	10	14	0.1455	0.583
	4	0	0	6	12	14	0.1275	0.538
	3	1	0	6	13	12	0.1261	0.480
	5	0	0	2	15	12	0.1143	0.444
	4	0	1	0	18	9	0.1111	0.333
23	1	0	0	19	3	21	0.3678	0.875
	0	1	0	19	4	19	0.2875	0.826
	2	0	0	15	6	19	0.2033	0.760
	1	1	0	15	7	17	0.1829	0.708
	3	0	0	11	9	17	0.1501	0.654
	2	1	0	11	10	15	0.1429	0.600
	4	0	0	7	12	15	0.1250	0.556
	3	1	0	7	13	13	0.1231	0.500
	5	0	0	3	15	13	0.1114	0.464
	4	0	1	1	18	10	0.1073	0.357
24	1	0	0	20	3	22	0.3663	0.880
	0	1	0	20	4	20	0.2857	0.833
	2	0	0	16	6	20	0.2016	0.769
	1	1	0	16	7	18	0.1808	0.720
	3	0	0	12	9	18	0.1481	0.667
	2	1	0	12	10	16	0.1405	0.615
	4	0	0	8	12	16	0.1228	0.571
	3	1	0	8	13	14	0.1204	0.519
	5	0	0	4	15	14	0.1089	0.483
	6	0	0	0	18	12	0.1010	0.400

Table 2.2 (continued)

No. of blocks (b)	D_1 $b_1 = 4$ $\lambda_0^{(1)} = 3$ $\lambda_1^{(1)} = 2$	D_2 $b_2 = 4$ $\lambda_0^{(2)} = 4$ $\lambda_1^{(2)} = 0$	D_3 $b_3 = 6$ $\lambda_0^{(3)} = 6$ $\lambda_1^{(3)} = 1$	D_4 $b_4 = 1$ $\lambda_0^{(4)} = 0$ $\lambda_1^{(4)} = 1$	λ_0	λ_1	τ^2	ρ
25	1	0	0	21	3	23	0.3649	0.885
	0	1	0	21	4	21	0.2841	0.840
	2	0	0	17	6	21	0.2000	0.778
	1	1	0	17	7	19	0.1790	0.731
	0	2	0	17	8	17	0.1645	0.680
	3	0	0	13	9	19	0.1464	0.679
	2	1	0	13	10	17	0.1385	0.630
	4	0	0	9	12	17	0.1208	0.586
	3	1	0	9	13	15	0.1180	0.536
	5	0	0	5	15	15	0.1067	0.500
	4	1	0	5	16	13	0.1066	0.448
	6	0	0	1	18	13	0.0984	0.419
26	1	0	0	22	3	24	0.3636	0.889
	0	1	0	22	4	22	0.2826	0.846
	2	0	0	18	6	22	0.1986	0.786
	1	1	0	18	7	20	0.1773	0.741
	0	2	0	18	8	18	0.1625	0.692
	3	0	0	14	9	20	0.1448	0.690
	2	1	0	14	10	18	0.1366	0.643
	4	0	0	10	12	18	0.1190	0.600
	3	1	0	10	13	16	0.1159	0.552
	5	0	0	6	15	16	0.1046	0.516
	4	1	0	6	16	14	0.1042	0.467
	6	0	0	2	18	14	0.0961	0.438
	5	0	1	0	21	11	0.0938	0.344
27	1	0	0	23	3	25	0.3625	0.893
	0	1	0	23	4	23	0.2813	0.852
	2	0	0	19	6	23	0.1973	0.793
	1	1	0	19	7	21	0.1758	0.750
	0	2	0	19	8	19	0.1607	0.704
	3	0	0	15	9	21	0.1434	0.700
	2	1	0	15	10	19	0.1349	0.655
	4	0	0	11	12	19	0.1174	0.613
	3	1	0	11	13	17	0.1140	0.567
	5	0	0	7	15	17	0.1028	0.531
	4	1	0	7	16	15	0.1020	0.484
	6	0	0	3	18	15	0.0940	0.455
	5	0	1	1	21	12	0.0911	0.364

Table 2.3

Optimal Designs[1/] and Associated Confidence Coefficient (P) as a Function of

b and d/σ for $\dot{p} = 4$, $k = 4$

No. of blocks (b)	d/σ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
4	1,0 0,0 0.2119	1,0 0,0 0.2602	1,0 0,0 0.3134	1,0 0,0 0.3706	1,0 0,0 0.4306	1,0 0,0 0.4921	1,0 0,0 0.5535	1,0 0,0 0.6134	1,0 0,0 0.6705	1,0 0,0 0.7237
5	1,0 0,1 0.2480	1,0 0,1 0.3013	1,0 0,1 0.3592	1,0 0,1 0.4203	1,0 0,1 0.4833	1,0 0,1 0.5465	1,0 0,1 0.6084	1,0 0,1 0.6675	1,0 0,1 0.7226	1,0 0,1 0.7726
6	1,0 0,2 0.2751	1,0 0,2 0.3318	1,0 0,2 0.3926	1,0 0,2 0.4559	1,0 0,2 0.5203	1,0 0,2 0.5840	1,0 0,2 0.6455	1,0 0,2 0.7032	1,0 0,2 0.7561	1,0 0,2 0.8034
7	1,0 0,3 0.2965	1,0 0,3 0.3555	1,0 0,3 0.4181	1,0 0,3 0.4828	1,0 0,3 0.5479	1,0 0,3 0.6115	1,0 0,3 0.6722	1,0 0,3 0.7286	1,0 0,3 0.7796	1,0 0,3 0.8245
8	1,0 0,4 0.3139	1,0 0,4 0.3746	1,0 0,4 0.4385	1,0 0,4 0.5040	1,0 0,4 0.5693	2,0 0,0 0.6415	2,0 0,0 0.7186	2,0 0,0 0.7862	2,0 0,0 0.8429	2,0 0,0 0.8885
9	1,0 0,5 0.3283	1,0 0,5 0.3904	1,0 0,5 0.4552	1,0 0,5 0.5212	2,0 0,1 0.5883	2,0 0,1 0.6713	2,0 0,1 0.7464	2,0 0,1 0.8110	2,0 0,1 0.8640	2,0 0,1 0.9057
10	1,0 0,6 0.3406	1,0 0,6 0.4037	1,0 0,6 0.4692	1,0 0,6 0.5356	2,0 0,2 0.6127	2,0 0,2 0.6948	2,0 0,2 0.7679	2,0 0,2 0.8297	2,0 0,2 0.8797	2,0 0,2 0.9181

[1/] The matrix in each cell is $\left\{\begin{matrix} \hat{f}_1, \hat{f}_2 \\ \hat{f}_3, \hat{f}_4 \end{matrix}\right\}$ where $\hat{D} = \bigcup_{i=1}^{4} \hat{f}_i D_i$ with $b = \sum_{i=1}^{4} \hat{f}_i b_i$ is the optimal design for the given value of b and d/σ.

Table 2.3 (continued)

No. of blocks	d/σ									
(b)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
11	1,0 0,7 0.3513	1,0 0,7 0.4151	1,0 0,7 0.4811	1,0 0,7 0.5477	2,0 0,3 0.6329	2,0 0,3 0.7138	2,0 0.3 0.7850	2,0 0,3 0.8444	1,0 1,1 0.8952	1,0 1,1 0.9333
12	1,0 0,8 0.3605	1,0 0,8 0.4250	1,0 0.8 0.4915	2,0 0,4 0.5623	3,0 0,0 0.6515	3,0 0,0 0.7433	3,0 0,0 0.8201	3.0 0,0 0.8802	3,0 0,0 0.9242	3,0 0,0 0.9545
13	1,0 0,9 0.3687	1,0 0,9 0.4337	1,0 0,9 0.5005	2,0 0,5 0.5771	3,0 0,1 0.6721	3,0 0,1 0.7620	3,0 0,1 0.8360	3,0 0,1 0.8928	3,0 0,1 0.9336	3,0 0,1 0.9611
14	1,0 0,10 0.3761	1,0 0,10 0.4415	1,0 0.10 0.5085	2,0 0,6 0.5899	3,0 0,2 0.6895	3,0 0,2 0.7776	3,0 0,2 0.8490	3,0 0,2 0.9030	2,0 1,0 0.9414	2,0 1,0 0.9675
15	1,0 0.11 0.3826	1,0 0,11 0.4484	1,0 0,11 0.5157	3,0 0,3 0.6041	3,0 0,3 0.7044	3,0 0,3 0.7907	3,0 0,3 0.8598	2.0 1.1 0.9139	2,0 1,1 0.9507	2,0 1.1 0.9735
16	1,0 0,12 0.3886	1,0 0,12 0.4547	1.0 0,12 0.5221	3,0 0,4 0.6179	4,0 0,0 0.7237	4,0 0,0 0.8151	4,0 0,0 0.8844	4,0 0,0 0.9326	4,0 0,0 0.9633	4,0 0,0 0.9813
17	1,0 0,13 0.3940	1,0 0,13 0.4604	2,0 0,9 0.5290	3,0 0,5 0.6302	4,0 0,1 0.7384	4.0 0,1 0.8275	4,0 0,1 0.8940	4,0 0,1 0.9393	4.0 0.1 0.9676	4,0 0,1 0.9839

Table 2.3 (continued)

No. of blocks (b)	d/σ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
18	1,0 0,14 0.3990	1,0 0,14 0.4656	2,0 0,10 0.5372	4,0 0,2 0.6436	4,0 0,2 0.7513	4,0 0,2 0.8382	4,0 0,2 0.9020	3,0 1,0 0.9456	3,0 1,0 0.9724	3,0 1.0 0.9871
19	1,0 0,15 0.4036	1,0 0,15 0.4704	2,0 0,11 0.5447	4,0 0,3 0.6562	4,0 0,3 0.7626	4,0 0,3 0.8474	3,0 1,1 0.9105	3,0 1.1 0.9522	3,0 1,1 0.9765	3,0 1,1 0.9893
20	1,0 0,16 0.4078	1,0 0.16 0.4748	3,0 0,8 0.5521	4,0 0,4 0.6675	5,0 0,0 0.7802	5,0 0,0 0.8665	5,0 0,0 0.9256	5,0 0,0 0.9619	5,0 0,0 0.9821	5,0 0,0 0.9923
21	1,0 0,17 0.4117	1,0 0,17 0.4789	3,0 0,9 0.5605	5,0 0,1 0.6801	5,0 0,1 0.7912	5,0 0,1 0.8750	5,0 0,1 0.9314	5,0 0,1 0.9656	5,0 0,1 0.9842	5,0 0,1 0.9934
22	1,0 0,18 0.4154	1,0 0,18 0.4827	3,0 0,10 0.5682	5,0 0,2 0.6914	5,0 0.2 0.8009	5,0 0,2 0.8824	4.0 1,0 0.9370	4,0 1,0 0.9698	4,0 1,0 0.9868	4,0 1,0 0.9948
23	1,0 0,19 0.4188	1,0 0,19 0.4862	4,0 0,7 0.5758	5,0 0,3 0.7017	5,0 0,3 0.8096	4,0 1,1 0.8892	4,0 1,1 0.9428	4,0 1,1 0.9733	4,0 1,1 0.9886	4,0 1,1 0.9956
24	1,0 0,20 0.4220	1,0 0,20 0.4895	4,0 0,8 0.5840	6,0 0,0 0.7133	6,0 0,0 0.8249	6,0 0,0 0.9034	6,0 0,0 0.9520	6,0 0,0 0.9785	6,0 0,0 0.9913	6,0 0.0 0.9968

Table 2.3 (continued)

No. of blocks (b)	d/σ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
25	1,0 0,21 0.4250	1,0 0,21 0.4926	4,0 0,9 0.5916	6.0 0,1 0.7235	6,0 0,1 0.8332	6,0 0,1 0.9093	6,0 0,1 0.9557	6,0 0,1 0.9805	6,0 0,1 0.9923	6,0 0,1 0.9972
26	1,0 0,22 0.4278	1,0 0,22 0.4955	5,0 0,6 0.5991	6,0 0,2 0.7328	6,0 0,2 0.8406	6,0 0,2 0.9145	5,0 1,0 0.9598	5,0 1.0 0.9831	5,0 1,0 0.9937	5,0 1,0 0.9979
27	1,0 0,23 0.4305	1,0 0,23 0.4983	5,0 0,7 0.6071	6,0 0,3 0.7413	6,0 0,3 0.8473	5,0 1,1 0.9203	5,0 1,1 0.9633	5,0 1,1 0.9850	5,0 1,1 0.9945	5,0 1,1 0.9982
28	1,0 0,24 0.4330	1,0 0,24 0.5009	5,0 0,8 0.6145	7,0 0,0 0.7525	7,0 0,0 0.8603	7,0 0,0 0.9301	7,0 0,0 0.9690	7,0 0,0 0.9878	7,0 0,0 0.9957	7,0 0,0 0.9987
29	1,0 0,25 0.4354	1,0 0,25 0.5033	6,0 0,5 0.6219	7,0 0,1 0.7609	7,0 0,1 0.8667	7,0 0,1 0.9342	7,0 0,1 0.9713	7,0 0,1 0.9889	7,0 0,1 0.9962	7,0 0,1 0.9989
30	1,0 0,26 0.4377	1,0 0,26 0.5057	6,0 0,6 0.6294	7,0 0,2 0.7686	7,0 0,2 0.8724	6,0 1,0 0.9384	6,0 1,0 0.9742	6,0 1,0 0.9905	6,0 1,0 0.9969	6,0 1,0 0.9991
31	1,0 0,27 0.4398	1,0 0,27 0.5079	6,0 0,7 0.6365	7,0 0,3 0.7756	7.0 0,3 0.8776	6,0 1,1 0.9425	6,0 1,1 0.9764	6,0 1,1 0.9915	6,0 1,1 0.9973	6,0 1,1 0.9993

Table 2.4

Optimal Design[1/] to Achieve a Specified Confidence Coefficient

as a Function of d/σ

for $p = 4$, $k = 4$

Confidence Coefficient	d/σ									
$(1-\alpha)$	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.99	b=470 116,0 1,0	b=118 28,0 1,0	b=53 13,0 0,1	b=30 6,0 1,0	b=20 5,0 0,0	b=14 2,0 1,0	b=11 1,0 1,1	b=8 2,0 0,0	b=7 0,0 1,1	b=6 0,0 1,0
0.95	b=290 71,0 1,0	b=73 18,0 0,1	b=33 8,0 0,1	b=19 3,0 1,1	b=12 3,0 0,0	b=9 2,0 0,1	b=7 0,0 1,1	b=5 1,0 0,1	b=4 1,0 0,0	b=4 1,0 0,0
0.90	b=213 53,0 0,1	b=54 13,0 0,2	b=24 6,0 0,0	b=14 3,0 0,2	b=9 2,0 0,1	b=7 0,0 1,1	b=5 1,0 0,1	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0
0.85	b=168 42,0 0,0	b=43 10,0 0,3	b=20 5,0 0,0	b=12 3,0 0,0	b=8 2,0 0,0	b=5 1,0 0,1	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0
0.80	b=136 34,0 0,0	b=35 8,0 0,3	b=16 4,0 0,0	b=9 2,0 0,1	b=6 1,0 0,2	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0
0.75	b=112 28,0 0,0	b=28 7,0 0,0	b=13 3,0 0,1	b=8 2,0 0,0	b=5 1,0 0,1	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0	b=4 1,0 0,0

[1/] The matrix in each cell is $\left\{\begin{matrix} \hat f_1, \hat f_2 \\ \hat f_3, \hat f_4 \end{matrix}\right\}$ where $\hat D = \bigcup_{i=1}^{4} \hat f_i D_i$ with $b = \sum_{i=1}^{4} \hat f_i b_i$ is the optimal design for the given value of $1 - \alpha$ and d/σ.

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[1] Bechhofer, R.E. and Tamhane, A.C. (1979a). Incomplete block designs for comparing treatments with a control (I): General theory. (Submitted for publication.)

[2] Bechhofer, R.E. and Tamhane, A.C. (1979b). Incomplete block designs for comparing treatments with a control (II): Optimal designs for $p = 2(1)6$, $k = 2$ and $p = 3$, $k = 3$. (Submitted for publication.)

[3] Bechhofer, R.E. and Tamhane, A.C. (1979c). Incomplete block designs for comparing treatments with a control (III): Optimal designs for $p = 4$, $k = 3$ and $p = 5$, $k = 3$. (Submitted for publication.)

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Multiple comparisons with a control, balanced treatment incomplete block (BTIB) designs, admissible designs, S-inadmissible designs, C-inadmissible designs, minimal complete class of generator designs, optimal designs.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The present paper continues the study of balanced treatment incomplete block (BTIB) designs initiated ~~in [1], [2], and [3]~~. This class of of designs was proposed for the problem of comparing simultaneously $p \geq 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p + 1$. A list of generator designs, the conjectured minimal complete class of generator designs, a catalog of admissible designs, and tables of optimal designs are given for $p = 4, k = 4$. Some comparisons are made with admissible designs for $p = 4$, $k = 3$. (over)

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[1] Bechhofer, R.E. and Tamhane, A.C. (1979a). Incomplete block designs for comparing treatments with a control (I): General theory. (Submitted for publication.)

[2] Bechhofer, R.E. and Tamhane, A.C. (1979b). Incomplete block designs for comparing treatments with a control (II): Optimal designs for $p = 2(1)6$, $k = 2$ and $p = 3$, $k = 3$. (Submitted for publication.)

[3] Bechhofer, R.E. and Tamhane, A.C. (1979c). Incomplete block designs for comparing treatments with a control (III): Optimal designs for $p = 4$, $k = 3$ and $p = 5$, $k = 3$. (Submitted for publication.)